The Simplest Optimizing Sticky Price Model? New ISLM*

1. Introduction

Recently several macroeconomists have begun to use a stylized model, based on dynamically optimizing behavior with sticky prices, that uses just two equations to analyze the effects of monetary and fiscal policy. The two equations bear some relation to a traditional ISLM equation and a Phillips curve. Here we show the kind of equilibrium model that can lead to such equations, and discuss the limitations of this kind of modeling.

2. The consumer

The representative consumer maximizes

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(U(C_t, 1-L_t)+\gamma\left(\frac{M_t}{P_t}\right)\right)\right] \tag{1}$$

subject to

$$C_t+\frac{B_t+M_t}{P_t}=\frac{W_t}{P_t}L_t+\frac{R_{t-1}B_{t-1}+M_{t-1}}{P_t}+\pi_t-\tau_t\,. \tag{2}$$

The consumer chooses time paths for C, L, B, and M, with the usual dating convention that choice variables dated t are allowed to depend on non-choice variables dated t and earlier only. The variable names should be largely self-explanatory. The variable π represents profit distributions from the representative firm, which is owned by the consumer, but non-tradeable; and τ is taxes. The term involving γ in the utility function is meant to represent the transactions value of money. It is a drawback of this setup that it implies no increase in the transactions value of money as the level of consumption increases, so long as real balances M/P stay constant. More natural would be to have transactions costs as a proportion of output stay constant as long as the ratio of M/P to C stays constant. But this more realistic specification would keep us from getting to the simple stylized model we are aiming at.

The government budget constraint is

$$\frac{B_t+M_t}{P_t}=\frac{R_{t-1}B_{t-1}+M_{t-1}}{P_t}-\tau_t\,, \tag{3}$$

so that in equilibrium

$$C_t = \frac{W_t}{P_t} L_t + \pi_t \,. \tag{4}$$

The consumer Euler equations are

$$\partial C: \qquad D_1 U_t = \lambda_t \tag{5}$$

$$\partial L: \qquad D_2 U_t = \lambda_t \frac{W_t}{P_t} \tag{6}$$

$$\partial M: \qquad \gamma' \left(\frac{M_t}{P_t} \right) = \frac{\lambda_t}{P_t} - \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] \tag{7}$$

$$\partial B: \qquad \frac{\lambda_t}{P_t} = R_t \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}} \right] \,. \tag{8}$$

3. The firm

The firm maximizes

$$E \left[\sum_{t=0}^{\infty} \beta^t \Phi_t \pi_t \right] \tag{9}$$

subject to

$$\mu: \qquad \pi_t = C_t - \frac{W_t}{P_t} L_t \tag{10}$$

$$\psi: \qquad C_t = F(L_t) - \frac{\xi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 \tag{11}$$

$$\nu: \qquad \frac{C_t}{\bar{C}_t} = \left(\frac{P_t}{\bar{P}_t} \right)^{-\theta} \,, \tag{12}$$

where the Greek letters at the left are the Lagrange multipliers we will associate with each constraint. The firm chooses time paths for π, C, L, and P. The stochastic discount factors Φ_t are taken by the firm as given by the market. It will turn out that in the simple, linearized special case we consider below these discount factors do not matter, so long as they are constant in steady state. Equation (10) defines profit distributions as what's left over from production after wage payments. Equation (11) defines production as a function of labor (no capital) less a charge for rapid proportional change in prices. Equation (12) defines the firm's demand curve, as derived from the Dixit-Stiglitz setup. In the firm's problem, C_t is interpreted as the firm's own output of the consumption good, while $\bar{C}_t$ is the aggregate output. In equilibrium, all firms will make the same choice and we will have $C_t \equiv \bar{C}_t$, but individual firms perceive a possibility of deviating, and we must account for this in solving their maximization problem.

FOC's for the firm are

$$\partial\pi: \qquad \Phi_t = \mu_t \tag{13}$$

$$\partial C: \qquad -\mu_t + \psi_t + \frac{\nu_t}{\bar{C}_t} = 0 \tag{14}$$

$$\partial L: \qquad \mu_t \frac{W_t}{P_t} = \psi_t F'_t \tag{15}$$

$$\begin{aligned} \partial P: \qquad & -\mu_t \frac{W_t L_t}{P_t^2} + \nu_t \theta \frac{P_t^{-\theta-1}}{\bar{P}_t^{-\theta}} + \psi_t \xi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{1}{P_{t-1}} \\ & - \beta \xi E_t \left[\psi_{t+1} \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t^2} \right] = 0 \,. \end{aligned} \tag{16}$$

Note that we use the same letters for C and L for the firm and consumer problems, as a way of implicitly imposing market clearing.

4. Analysis of the system of equations

We can solve to get rid of Lagrange multipliers in both consumer and firm problems. For the consumer, we get from (5)-(8)

$$\frac{D_2 U_t}{D_1 U_t} = \frac{W_t}{P_t} \tag{17}$$

$$\frac{D_1 U_t}{P_t} = \beta R_t E_t \left[\frac{D_1 U_{t+1}}{P_{t+1}} \right] \tag{18}$$

$$\frac{\gamma'_t}{P_t} = \frac{D_1 U_t}{P_t} - \beta E_t \left[\frac{D_1 U_{t+1}}{P_{t+1}} \right] \,. \tag{19}$$

For the firm, by using (13)-(15) in (16) and multiplying the result through by $F'_t P_t^2/(W_t \Phi_t)$, we arrive at the single equation

$$\begin{aligned} & -F'_t L_t + C_t \left(\frac{P_t F'_t}{W_t} - 1 \right) \theta \\ & \qquad + \xi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} - \beta \xi E_t \left[\left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{W_{t+1} F'_t \Phi_{t+1}}{W_t F'_{t+1} \Phi_t} \right] \,, \end{aligned} \tag{20}$$

where we have used the fact that in equilibrium $P_t = \bar{P}_t$ and $C_t = \bar{C}_t$. In order to allow some further simplification, we will consider the special case of linear production function, so that $F(L_t) = F'_t \cdot L_t$. This lets us conclude from (20) that in steady-state we will have (letting barred variables without subscripts indicate steady-state values)

$$\frac{\bar{W}}{\bar{P}\bar{F}'} = \frac{\theta}{1+\theta} \,, \tag{21}$$

i.e. a markup of marginal product of labor over real wage that depends on the elasticity of demand. Also, we will have $\bar{C} = \bar{F}'\bar{L}$, as the penalty term on price changes in (11) vanishes in deterministic steady state. (Note that this means we are considering only zero-inflation steady states.)

Log-linearizing (20) about deterministic steady state leads then (using lower case to indicate the log of a variable)

$$(1+\theta)\bar{C}\cdot(dp_t + df'_t - dw_t) + \xi(dp_t - dp_{t-1}) - \beta\xi(dp_{t+1} - dp_t) = \eta_p(t+1)\,, \qquad (22)$$

where $\eta_p(t)$ is an endogenous error term with the usual property for such terms that $E_t\eta_{t+1} = 0$. Note that the dc_t term that might have been expected drops out.

Before we proceed to linearize what this literature labels the "IS" curve, equation (18), we introduce one further simplification that makes L drop out of that equation: we assume U is additively separable in C and $1-L$. So log-linearized (18) becomes

$$dc_t = dc_{t+1} - \sigma^{-1}(dr_t - dp_{t+1} + dp_t) + \eta_c(t+1)\,, \qquad (23)$$

where $\sigma = -D_{11}\bar{U}\cdot\bar{C}/\bar{D}_1U$ is the rate of relative risk aversion at steady state consumption.

Log-linearizing (17) leads to

$$\sigma dc_t + \alpha d\ell_t = dw_t - dp_t\,, \qquad (24)$$

where α is a positive constant. Using also the linearized technology constraint (11),

$$dc_t = df'_t + d\ell_t\,, \qquad (25)$$

we can express (22) entirely in terms of dp, dc, dr, the exogenous technology disturbance df', and the endogenous error term:

$$(1+\theta)\bar{C}\cdot(-(\sigma+\alpha)dc_t + (1+\alpha)df'_t) + \xi(dp_t - dp_{t-1}) - \beta\xi(dp_{t+1} - dp_t) = \eta_p(t+1)\,. \qquad (26)$$

Note that (contrary to what I said in class Tuesday 4/6) this equation is written entirely in terms of inflation, with no terms in the price level. It may be helpful to intuition to note that *if* the inflation rate does not explode at the rate β^{-1} or faster, then the equation can be solved forward to express current inflation as a weighted sum of current and future dc, with positive weights, and technology shocks df', with negative weights. However, it is also important to recognize that dc and dp are jointly determined endogenous variables, so no conclusions about existence and uniqueness of stable solutions are possible by examining (26) in isolation.

5. Completing the system: Monetary policy

To keep the system down to two dimensions, let us postulate a monetary policy rule, in terms of deviations from steady state, of the form

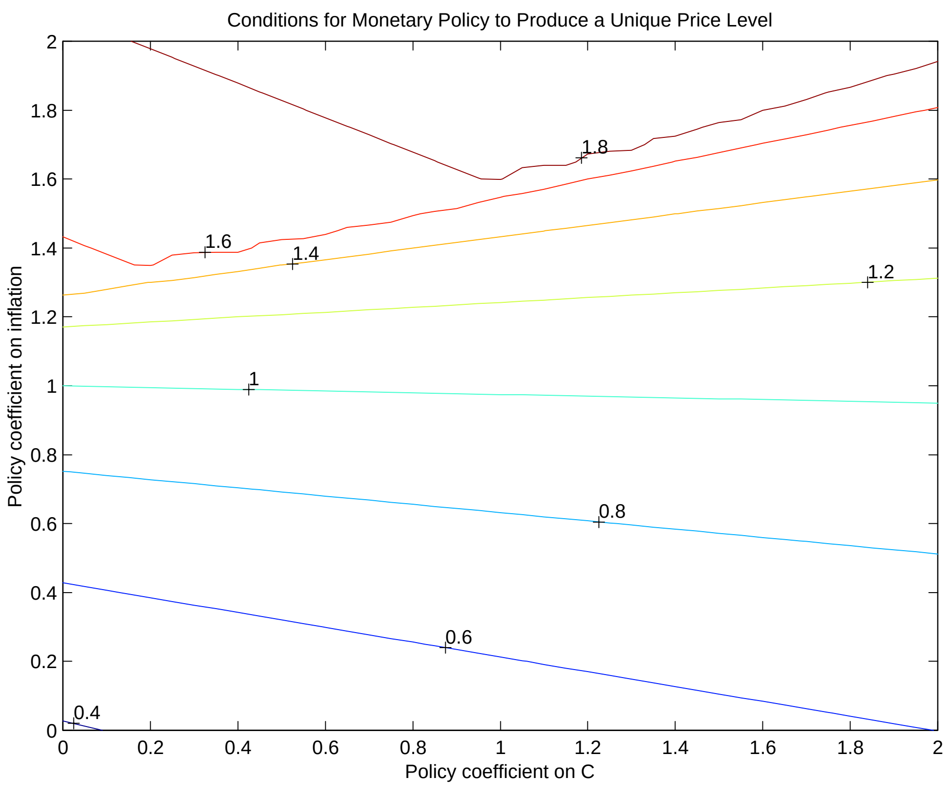
$$dr_t = \delta_0 dq_t + \delta_1 dc_t + \varepsilon_m(t)\,. \qquad (27)$$

Here we are introducing the notation $dq_t = dp_t - dp_{t-1}$ for the deviation of inflation from steady state. By omitting lags (an important omission for a serious study of the effects of monetary policy) we can just substitute for dr_t in (23) and arrive at the system, in matrix format,

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} dc_{t+1} \\ dq_{t+1} \end{bmatrix} = \begin{bmatrix} 1+\sigma^{-1}\delta_1 & \sigma^{-1}\delta_0 \\ -(\sigma+\alpha)\bar{C} & 1 \end{bmatrix} \begin{bmatrix} dc_t \\ dq_t \end{bmatrix} + \begin{bmatrix} 0 & \sigma^{-1} \\ \bar{C}\frac{1+\alpha}{\xi} & 0 \end{bmatrix} \begin{bmatrix} df'_t \\ \varepsilon_m(t) \end{bmatrix} + \begin{bmatrix} \eta_c(t+1) \\ \eta_p(t+1) \end{bmatrix} . \quad (28)$$

Using the standard notation of previous lectures and notes on linear rational expectations models, the matrices of coefficients in this equation are labeled Γ_0, Γ_1, and Ψ, with what is called Π in the notes (the coefficient of the η's) not appearing because it is the identity. We can expect to solve this system forward, eliminating the indeterminacy due to the presence of the two η's, if $A = \Gamma_0^{-1}\Gamma_1$ has all its eigenvalues greater than one in absolute value. A computational trick that helps limit the algebra here is to note that the x's that solve $\left|Ix - \Gamma_0^{-1}\Gamma_1\right| = 0$ are the same as those that solve $|\Gamma_0 x - \Gamma_1| = 0$. (Though inverting Γ_0 is not a major task,here.)

A contour plot of the absolute value of the minimum root as a function of δ_0 and δ_1 appears in the figure below. These were computed with $\beta = .95$, $\alpha = \sigma = 2$, and $\bar{C} = .5$.



Clearly the minimum root is larger than one in absolute value when δ_0 is above one. Large positive values of δ_1 make it possible for δ_0 to go just slightly below one. That $\delta_0 > 1$ should be required makes sense, as this is the condition that the monetary authority responds to inflation with a strong enough rise in the nominal interest rate to make the real rate increase.

Assuming the df_t' shocks are i.i.d., we can easily derive the solution for dc_t and dp_t in terms of the exogenous disturbances df_t' and $\varepsilon_m(t)$, by solving forward. Using the same parameter settings as for the graph above, with $\delta_0 = 1.1$ and $\delta_1 = .5$, we obtain as the matrix of impacts of exogenous disturbances

$$\begin{bmatrix} 0.3511 & -0.2128 \\ -0.7979 & -0.4255 \end{bmatrix} \tag{29}$$

This means that a surprise money contraction (ε_m increase) shrinks both consumption and inflation, while a surprise favorable productivity shock (df_t' increase) produces a rise in consumption and a fall in prices. This accords nicely with our informal ideas about how "demand" and "supply" shocks should affect the economy. But note that if df_t' and $\varepsilon_m(t)$ are i.i.d., the forward-solved model implies i.i.d. dq and dc, despite the price "stickiness". This does not match very well what we know about the data.

6. Limitations of the model

If the model had increasing costs in the investment goods industry or had adjustment costs on capital, internal to the firms, then we would find it necessary to include more lagged variables in the linearization. This would create more persistence in response to shocks.

The Keynesian econometric modeling tradition thought of wages as sticky, due to the institutional characteristics of wage bargaining, and of prices as responding in a relatively simple way to wages via markup rules. Statistical evidence for this view that stickiness in some sense originates in the labor market, with little delay in the reaction of prices to wages, is weak. However, there is certainly no strong support for the opposite view, that stickiness originates in pricing, with wages responding quickly to prices. The model discussed in these notes attributes all stickiness to price menu costs and has real wages adjusting quickly to clear markets. The implications of this model cannot easily be made to match the data in certain respects, particularly the cyclical behavior of real wages.

One could set up the model with stickiness in wages, just by having individual consumers face Dixit-Stiglitz demand curves for their particular types of labor and a menu cost for wage changes. The main reason this has not often been done is probably that there is no intuitively appealing fable, like the restaurant owner reprinting his menus, that helps make us comfortable with the idea of a monopolistically competitive worker facing wage-change menu costs. But that wages are indeed sticky is certainly plausible, and the idea that a worker, via length of search, faces a tradeoff between the level of wages he seeks and the fraction of time he will be working, is also plausible.

A few papers, e.g. the thesis of Jinill Kim and my paper "Stickiness" have combined wage and price stickiness, with monopolistic menu costs for both.

The model as it stands makes no use of the government budget constraint. Results we have obtained on existence and uniqueness of equilibrium are incomplete, therefore. Cases where the equilibrium looks indeterminate in the system as we have studied it here can turn out to be determinate under certain conditions on fiscal policy. Cases that appear to have a unique, determinate price level can turn out to imply non-existence of equilibrium under other conditions on fiscal policy. This leads us to ask more systematically how monetary and fiscal policy are related, the topic of the next section of the course.

7. Exercise

Consider the linearized model we discussed above, with the policy equation modified to the form

$$dr_t = \delta_0 dq_t + \delta_1 dc_t + \rho dr_{t-1} \varepsilon_m(t) \tag{30}$$

Use the same for parameters in other equations as were used in deriving the numerical results above, but now consider $\rho = .1$, $\rho = .9$, and $\rho = 1.1$. Is the conclusion still that there is a unique price level when $\delta_0 > 1$ and not when δ_0 is much below 1? Is there a systematic dependence of the critical value of δ_0 on ρ?